(41) Evaluate $\underset{x\to 0}{Lt}\left\{\frac{1}{2x^2}-\frac{1}{2x\tan x}\right\}$.

Ans.→ $\underset{x\to 0}{Lt}\left\{\frac{1}{2x^2}-\frac{1}{2x\tan x}\right\}\quad[\infty-\infty]$

$=\underset{x\to 0}{Lt}\left\{\frac{1}{2x^2}-\frac{\cos x}{2x\sin x}\right\}$

$=\underset{x\to 0}{Lt}\left\{\frac{\sin x - x\cos x}{2x^2\cdot \sin}\right\}\quad\left[\frac{0}{0}\right]$

Hence from L' Hospital Rule

$$=\underset{x\to 0}{Lt}\left\{\frac{\cancel{\cos x}-1\times\cancel{\cos x}+x\sin x}{4x\sin x+2x^2\cos x}\right\}\quad\left[\frac{0}{0}\right]$$

$$=\underset{x\to 0}{Lt}\left\{\frac{1\times\sin x+x\cos x}{4\times1\sin x+4x\cos x+4x\cos x-2x^2\sin x}\right\}\quad\left[\frac{0}{0}\right]$$

$$=\underset{x\to 0}{Lt}\left\{\frac{\cos x+1\times\cos x-x\sin x}{4\cos x+8\times1\cos x-8x\sin x-4x\sin x-2x^2\cos x}\right\}$$

$$=\underset{x\to 0}{Lt}\left\{\frac{\cos x+\cos x-x\sin x}{4\cos x+8\cos x-8x\sin x-4x\sin x-2x^2\cos x}\right\}$$

$$=\left[\frac{1+1+0}{4\times1+8+0-0-0}\right]=\frac{\cancel{2}}{\cancel{12}\,6}=\frac{1}{6}\text{ A}$$

(42) Evaluate $\lim_{x\to 0}(\sin x)^{\tan x}$.

Ans.→ Let $y=\underset{x\to 0}{Lt}(\sin x)^{\tan x}$

Taking log both sides, we have

$$\log_e y = \underset{x \to 0}{Lt}\ \tan x \cdot \log(\sin x)$$

$$\log_e y = \underset{x \to 0}{Lt}\ \frac{\log \sin x}{\cot x} \left[\frac{\infty}{\infty}\right]$$

Hence, from L' Hospital's Rule

$$\log_e y = \underset{x \to 0}{Lt}\ \frac{\frac{1}{\sin x} \times \cos x}{-\operatorname{cosec}^2 x}$$

$$= \underset{x \to 0}{Lt}\ \frac{\frac{\cos x}{\sin x}}{-1/\sin^2 x}$$

$$= \underset{x \to 0}{Lt}\ -\sin x \cdot \cos x$$

$$= 0 \times 1 = 0$$

$$\log_e y = 0$$

$$y = e^0 = 1 \text{ Ans.}$$

(43) Evaluate $\underset{x \to 0}{Lt}\ x^x$.

Ans. → Let $y = \underset{x \to 0}{Lt}\ x^x \ [0^0]$

Taking log both sides, we have,

$$\log_e y = \underset{x\to 0}{Lt}\; x \log x$$

$$= \underset{x\to 0}{Lt} \frac{\log x}{\frac{1}{x}} \left[\frac{\infty}{\infty}\right]$$

Hence from L' Hospital's Rule, we have

$$\log_e y = \underset{x\to 0}{Lt} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$\log_e y = \underset{x\to 0}{Lt} (-x) = 0$$

$\therefore \log_e y = 0$

$\therefore y = e^0 = 1$ Ans.

(44) Evaluate $\underset{x\to p}{Lt} (x-p)^{x-p}$.

Ans.→ Let $y = \underset{x\to p}{Lt} (x-p)^{x-p} \; [0^0]$

Taking log both sides, we have,

$$\therefore \log_e y = \underset{x\to p}{Lt} (x-p) \log (x-p)$$

$$= \underset{x\to p}{Lt} \frac{\log (x-p)}{\frac{1}{x-p}} \left[\frac{\infty}{\infty}\right]$$

Hence, from L' Hospital's Rule, we have

$$\log_e y = \lim_{x \to p} \frac{\cancel{\frac{1}{x-p}}}{-\frac{\cancel{1}}{(x-p)^{\cancel{2}}}}$$

$$\log_e y = \lim_{x \to p} -(x-p) = 0$$

$$= \quad \therefore \log_e y = 0$$

$$\therefore y = e^0 = 1 \text{ Ans.}$$

(45) Evaluate $\lim_{x \to \frac{\pi}{2}} (\cos x)^{\cos x}$.

Ans. → Let $y = \lim_{x \to \frac{\pi}{2}} (\cos x)^{\cos x} \quad [0^0]$

Taking log both sides, we have

$$\therefore \log_e y = \lim_{x \to \frac{\pi}{2}} \cos x \cdot \log \cos x$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\log \cos x}{\frac{1}{\cos x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\log \cos x}{\sec x} \left[\frac{\infty}{\infty}\right]$$

Hence, from L' Hospital's Rule, we hav

$$\log_e y = \lim_{x \to \frac{\pi}{2}} \frac{\frac{1}{\cos x} \cdot (-\sin x)}{\sec x \cdot \tan x}$$

$$\log_e y = \lim_{x \to \frac{\pi}{2}} \frac{-\frac{\sin \cancel{x}}{\cos \cancel{x}}}{\frac{1}{\cos x} \times \frac{\sin \cancel{x}}{\cos \cancel{x}}}$$

$$= -\lim_{x \to \frac{\pi}{2}} \cos x = 0$$

$\therefore \log_e y = 0$

$\therefore y = e^0 = 1$ Ans.